A structural equilibrium model for long-term power planning in a liberalized electricity market

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- **Long-term generation planning problem:**
  - To find the optimal generation policies, in order to provide indications and target values for the short-term problem
  - To schedule fuel procurement
  - Used for budgeting purposes, and to plan generation expansion

Introduction

- **Long-term generation planning problem:**
  - To find the optimal generation policies, in order to provide indications and target values for the short-term problem
  - To schedule fuel procurement
  - Used for budgeting purposes, and to plan generation expansion
- **Issues:**
  - Environmental (eg. basins levels, emissions are regulated by the Kyoto protocol)
  - Technical (eg. units outages, LDC matching, fuel availability limits)
  - Competition (the industry has been deregulated)
  - Economic (eg. change of fuel prices)
  - Operational (pool type market, all generation goes to pool)

Work’s outline

- The offer side of an electricity market is modeled considering that the demand is exogenous and its load duration curve (LDC) can be predicted.
- Given the price seasonalities, a function is considered that gives the price in terms of the load duration.
- Other characteristics:
  - Use of the Bloom and Gallant formulation
  - Game Theory approach
  - Inflow scenarios

The electricity market-price

The market price is determined by various factors:

- Exogenous
  - Economic situation
  - Input fuel prices
  - Weather conditions
  - Demand
- Endogenous
  - Companies’ selling strategies
  - Companies’ generating strategies

The demand is quite inelastic but have hourly and weakly seasonalities. Furthermore it is influenced by the weather conditions. The price dynamic is heavily influenced by the these seasonalities.

Average market price model (1/2)

First we introduce a model for the average price: Given

- A set of available technologies
  $G = \{$nuclear, hydro, coal, fuel, combined cycle, special regime$\}$
- The average daily generation in $\underline{i^{th}}$ period ($e_{g,i}$), using the technology $g \in G$.
- A function f^d, that models the portion of the price that can be predicted in a deterministic way

The following model can be formulated for the average market price:

$$p_i = f_i^d + X_i + \sum_{g \in G} b_g e_{g,i} \tag{1}$$

Average market price model (2/2)

Where

- b_g are the coefficients that link the generation to the price
- X_i, the unobservable stochastic process, follows the following dynamic:

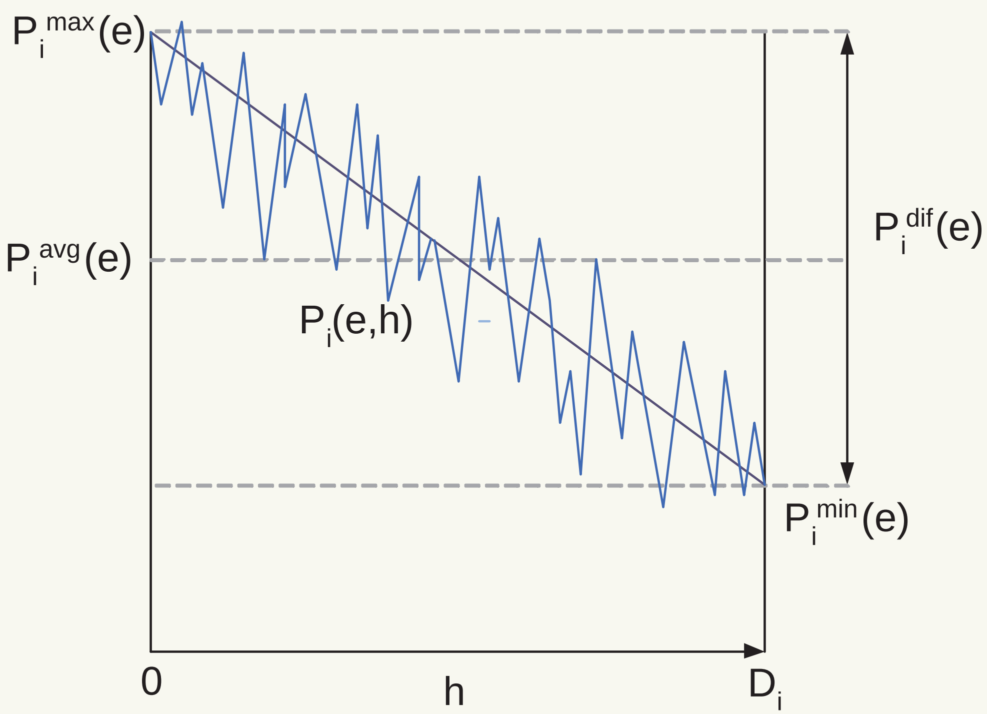
$$X_i = (1-\kappa)X_{i-1}\Delta t_i + \sigma_x\sqrt{\Delta t_i}\epsilon, \tag{2}$$

with,

- κ, the mean reversion rate
- σ_x, the unobservable process standard deviation
- ϵ, the noise shock $\in \mathcal{N}(0,1)$

Market price : modeling the price variability (1/2)

Looking at the historical series, when reordering the the prices by decreasing load power it can be observed a positive correlation between load and prices. For simplicity it is assumed that the prices change linearly with the load duration, even though more complicated dependencies can be observed.



Market price : modeling the price variability (2/2)

With these settings (linear dependence w.r.t the load), the price in terms of the load duration h and the generation, is expressed as follows:

$$\widetilde{p}_i(e,h) = p_i^{avg}(e_i) + p_i^{dif}(e_i)/2 - \frac{p_i^{dif}(e_i)h}{D_i} \qquad (3)$$

where

$$\begin{cases} \widehat{p}_i^{avg}(e_i) &= f_i^d + X_i^{avg} + \sum_{g\in G} b_g^{avg} e_{g,i} \\ \widehat{p}_i^{diff}(e_i) &= C + X_i^{dif} + \sum_{g\in G} b_g^{dif} e_{g,i}, \end{cases} \qquad (4)$$

with D_i, the duration of the $i^{\underline{th}}$ interval, and C the mean difference between maximum and minimum price.
where the two stochastic processes are

$$\begin{cases} X_i^{avg} &= (1-\kappa^{avg})X_{i-1}^{avg} + \sigma_X^{avg}\epsilon \\ X_i^{dif} &= (1-\kappa^{dif})X_{i-1}^{dif} + \sigma_X^{dif}\epsilon \end{cases}$$

Concepts: Game Theory

In a game with K players, let:

- $\mathbf{x} = (x_1, ..., x_K) \in X$ be the joint decision vector
- $r_k(\mathbf{x})$ be the profit function of the player k
- given $\mathbf{x} = (x_1, ..., x_K) \in X$ and $\mathbf{y} = (y_1, ..., y_K) \in X$, two joint decision vectors, $(y_k|\mathbf{x}) \in X$ is defined as the vector where all the companies $s \neq k$ play x_s, while the agent k plays y_k:

$$(y_k|\mathbf{x}) := (x_1, ..., x_{k-1}, y_k, x_{k+1}, ..., x_K). \tag{5}$$

- A game is called constrained if the players' action sets are constrained by the other players actions.
- A point $\mathbf{x}^*$ is called *Nash equilibrium* point if, for each k, the following holds:

$$r_k(\mathbf{x}^*) = \max_{(x_k|\mathbf{x}^*)\in X} r_k(x_k|\mathbf{x}^*).$$

Concepts: The NIRA Algorithm (1/2)

- The NIRA algorithm, exposed in Uryasev and Y. [1994] and Krawczyk and Uryasev [2000], has been introduced in the electricity market literature in Contreras et al. [2004]. The Nikaido-Isoda function, presented in Nikaido and Isoda [1955], is defined as follows:

$$\Psi(\mathbf{x}, \mathbf{y}) := \sum_{k=1}^{K} \left(r_k(y_k|\mathbf{x}) - r_k(\mathbf{x}) \right) \tag{6}$$

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- The Nikaido-Isoda function permits to give an equivalent formulation of a Nash equilibrium as an optimization problem: $\mathbf{x}^*$ is a Nash equilibrium if:

$$\max_{\mathbf{y} \in E} \Psi(\mathbf{x}^*, \mathbf{y}) = 0.$$

Concepts: The NIRA Algorithm (2/2)

making use of the *optimal response* function Z defined as follows:

$$Z(\mathbf{x}) := \arg\max_{\mathbf{y} \in X} \Psi(\mathbf{x},\mathbf{y}). \tag{7}$$

The algorithm updating rule is the following:

$$\mathbf{x}^{s+1} = (1 - \widetilde{\alpha}^s)\mathbf{x}^s + \widetilde{\alpha}^s Z(\mathbf{x}^s),$$

given the starting point $\mathbf{x}^0$ and a succession of scalars $\{\widetilde{\alpha}^s\}$, with $0 < \widetilde{\alpha}^s < 1$. Under conditions likely to be satisfied the equilibrium point is reached.

Concepts: The equivalent load method

- Given a set of generators Ω, with $N = |\Omega|$ and $j \in \Omega$ a generator, defined by its capacity c_j, outage probability q_j, and generating cost v_j, the expected generation can be calculated using the load survival function (LSF) $S(x)$:

$$e_j = D(1 - q_j) \int_0^{c_j} S(x)dx \,, \tag{8}$$

with D the period length.

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- Given $S_\Gamma(x)$ the LSF after a subset $\Gamma \subset \Omega$ of units has already been loaded, in Bálériaux et al. [1967] is introduced the following convolution formula

$$S_{\Gamma \cup j} = q_j S_\Gamma(x) + (1 - q_j) S_\Gamma(x + c_j), \tag{9}$$

  which expresses the change to the LSF caused by loading the $j^{\underline{th}}$ unit.

Concepts: The Bloom and Gallant Formulation

- The expected unsupplied energy is calculated as

$$w(\Gamma) = D \int_0^{p_{max}} S_\Gamma(x) dx$$

- Given E, the total energy of the Load Duration Curve (LDC), in Bloom and Gallant [1994] is established that, in order for the expected energies e_j, with $j \in \Omega$ to match the LDC, a set of linear inequality constraints, called load matching constraints (lmcs), must be satisfied:

$$\sum_{j \in \Gamma} e_j \leq E - w(\Gamma), \forall \Gamma \subset \Omega. \tag{10}$$

  There is an exponential number of such constraints.

- With the B&G model is also possible to include other (non-load matching) constraints to the probabilistic cost models.

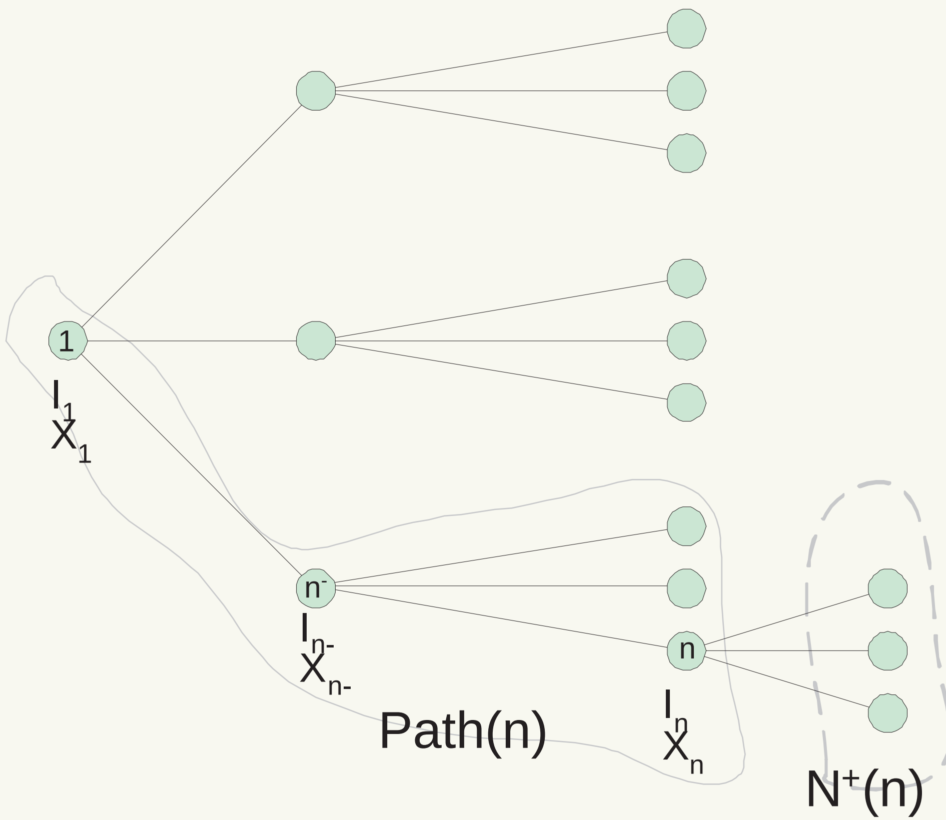
Scenario Discretization

The uncertainties to be discretized are:

- The inflows (I)
- The unobservable price factors (X)

we use a node notation instead of a scenario notation in order to save the non-anticipativity constraints. A node n, has the following properties:

- its predecessor n^-
- its time step t_n
- the value of the uncertainties (I_n, ϵ_n)



Inflows scenarios (I)

- Modeled as a Markov chain
- 3 states: *Dry*, *Normal*, and *Wet*. Each state is associated to a level of inflows.
- The state transition probabilities have been assigned empirically, considering that weather trends can change every 6-8 months.

Unobservable scenarios (X^{avg}, X^{dif})

- the first two conditional moments of X_i^* ($* = \{avg, dif\}$) can be calculated with the following expressions:

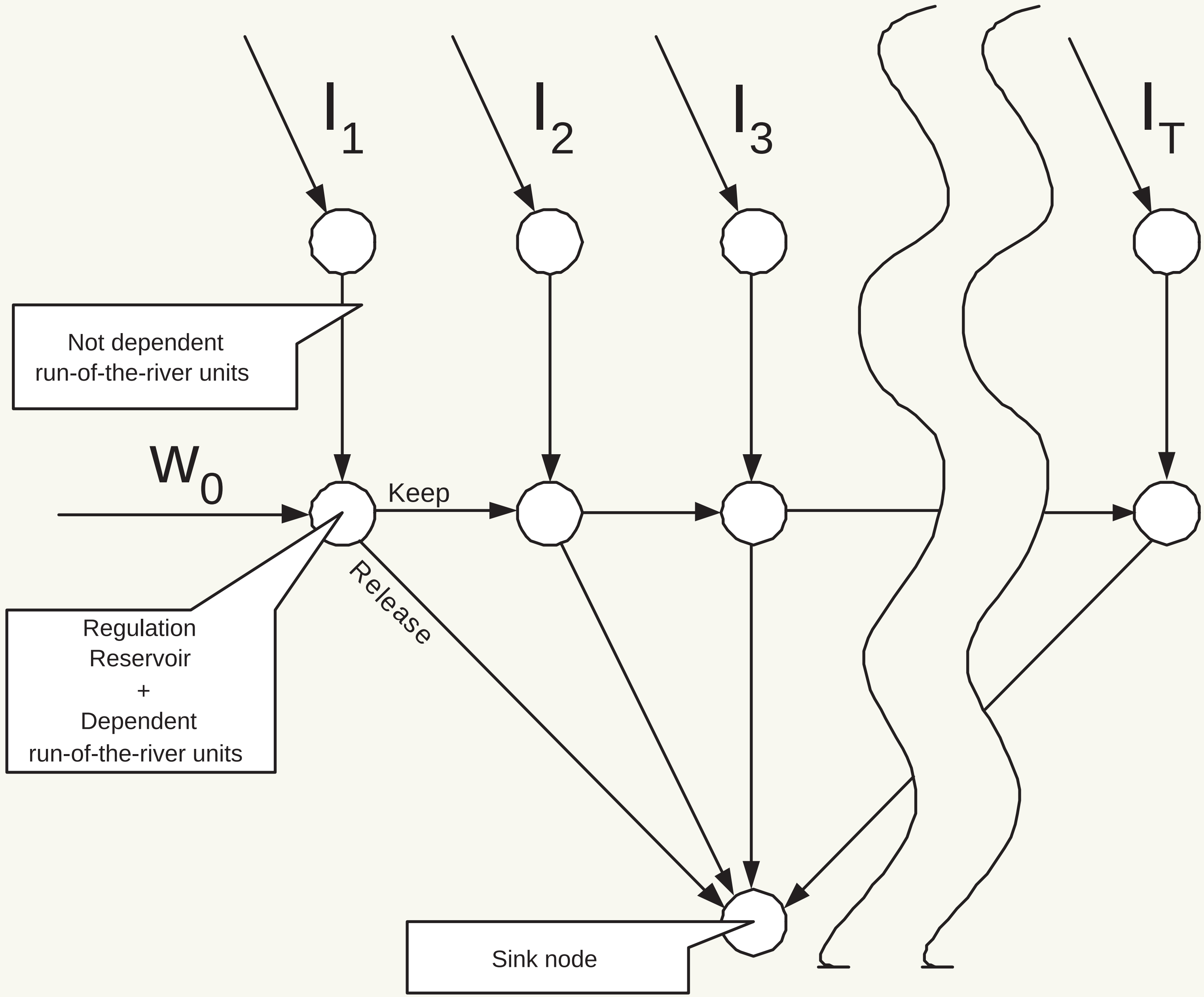
$$\mu(X_{i+1}^*|X_i^*) = X_i^*(1 - k^*\Delta t_i)$$

$$\sigma(X_{i+1}^*|X_i^*) = \sigma_{X^*}\sqrt{\Delta t_i}$$

- In order to generate a binomial tree that matches the mean and the standard deviation of the process, fixing the transition probabilities to 1/2, Given X_n^*, it can be obtained that the two outgoing nodes ($X_{n_1}^*, X_{n_2}^*$) have the following outcome:

$$\begin{cases} X_{n_1}^* = \mu(X_{t_n+1}^*|X_n^*) + \sigma(X_{t_n+1}^*|X_n^*) \\ X_{n_2}^* = \mu(X_{t_n+1}^*|X_n^*) - \sigma(X_{t_n+1}^*|X_n^*) \end{cases}$$

Hydrogeneration balance model



Hydrogeneration balance model

- The state variable $w_{k,n}$ that gives the remaining basin energy, can be expressed as a function of the natural inflows stochastic process I_n (expressed in MWh) and of the hydrogeneration energy:

$$w_{k,n} = \begin{cases} w_{k,n^-} - \sum_{j \in H \cap \Omega_k} e_{j,n^-} + s_k I_n, & \text{if } t_n > 1 \\ s_k(W^0 + I_n), & \text{otherwise} \end{cases}, \forall k, \forall n. \quad (11)$$

- s_k represents the $k^{\underline{th}}$ company hydrogeneration capacity share.

Reduced lmcs constraint set

- Given that considering all the load matching constraints (10), leads to an exponential number (with respect to the considered units) of linear inequalities, only a sub set of them will be considered in practice.
- it can be done using an heuristic as in Pagès and Nabona [2006].
- it can be chosen a lmcs subset a priori: we have considered all the subsets that can be built subtracting from Ω one and two units:

$$\begin{aligned} \Gamma_1 &= \{\mathit{\Gamma} \subset \mathit{\Omega} \mid \mathit{\Gamma} = \mathit{\Omega} \backslash \{j\} \text{ , } \forall j \in \mathit{\Omega}\} \\ \Gamma_2 &= \{\mathit{\Gamma} \subset \mathit{\Omega} \mid \mathit{\Gamma} = \mathit{\Omega} \backslash \{j,k\} \text{ , } \forall (j,k) \in \mathit{\Omega}^2, k > j\} \end{aligned} \quad (12)$$

, in addition to the single unit upper limits.

The profit function (1/2)

- The profit generated by the $j^{\underline{th}}$ unit in the tree node n is:

$$
\begin{aligned}
r_{j,n} &= \int_0^{e_{j,n}/c_j} c_j(\widetilde{p}_n(h,e) - v_j)dh \\
&= e_{j,n}(p^{avg}(e) + p^{dif}(e)/2 - v_j) + \frac{p^{dif}(e)e_{j,n}^2}{2p_j D_{t(n)}}
\end{aligned}
$$

- Substituting the expressions of $p^{avg}(e)$ and $p^{dif}(e)$, leads to the following cubic polynomial expression:

$$
\begin{aligned}
r_{j,n} = \; & e_{j,n}(f^d(t(n)) + X_n^{avg} + (C + X_n^{dif} - v_j)/2) \\
+ \; & e_{j,n}(\sum_{g \in G}(q_g^{avg} + q_g^{dif}/2)e_{g,n}) \\
+ \; & e_{j,n}^2(C + X_n^{dif})/(2p_j D_{t(n)}) \\
+ \; & e_{j,n}^2 \sum_{g \in G} q_g^{dif} e_{g,n}/(2p_j D_{t(n)})
\end{aligned}
$$

The profit function (2/2)

- With $r_{k,n}(e) = \sum_{j \in \Omega_k} r_{j,n}$ it is indicated the profit of the company k at node n.
- The System Operator, who is responsible for matching the demand by buying external energy, can be considered an additional player (the $(K+1)^{\text{th}}$).
- Assuming that the external energy has a fixed price, the following profit function can be derived:

$$r_{K+1,n}(e) = -p_{ext} e_{ext,n}$$

The two model formulations

two different behavior assumptions are considered :

- Sum of profit:
  - models the situation where the companies' generation planning is regulated by an external entity
  - used as benchmark
  - used as a starting solution for the NIRA algorithm
- Nash equilibrium:
  - models the situation where each player tries to maximize only his own profit.

Sum of profits formulation

The objective function (the maximization of the expected value of the sum of companies profit), is given by the following expression, representing a stochastic programming model in extensive form:

$$\underset{e}{\text{maximize}} \quad \sum_{k=1}^{K} \sum_{n \in \mathcal{N}} \pi_n r_{k,n}(e) - \sum_{n \in \mathcal{N}} \pi_n p_{ext} e_{ext,n} \tag{13}$$

$$\text{subject to} \quad \sum_{j \in \Gamma} e_{j,n} \leq E_{t_n} - w(\Gamma),\ \forall \Gamma \in \overline{\Gamma}_1 \cup \overline{\Gamma}_2 \tag{14}$$

$$\sum_{j \in \Omega^{\text{hydro}} \cap \Omega_k} e_{j,n} \leq w_{k,n},\ \forall k, \forall n \tag{15}$$

$$\sum_{j \in \Gamma} e_{j,n} + e_{ext,n} = E_{t_n},\ \forall n \tag{16}$$

$$\sum_{j \in \Omega^{\text{sp.reg}} \cap \Omega_k} e_{j,n} \leq R_{t_n},\ \forall k, \forall n \tag{17}$$

$$e \geq \underline{0} \tag{18}$$

Equilibrium formulation

The objective function is the particular case of (6), where the e' plays the role of **y**:

$$\underset{e}{\text{maximize}} \quad \sum_{k=1}^{K+1} \sum_{n \in \mathcal{N}} \pi_n \big(r_{k,n}(e'|e) - r_{k,n}(e)\big) \tag{19}$$

$$\text{subject to} \quad \sum_{j \in \Gamma} e'_{j,n} \leq E_{t_n} - w(\Gamma),\ \forall \Gamma \in \overline{\Gamma}_1 \cup \overline{\Gamma}_2 \tag{20}$$

$$\sum_{j \in \Omega^{\text{hydro}} \cap \Omega_k} e'_{j,n} \leq w_{k,n},\ \forall k, \forall n \tag{21}$$

$$\sum_{j \in \Gamma} e'_{j,n} + e'_{ext,n} = E_{t_n},\ \forall n \tag{22}$$

$$\sum_{j \in \Omega^{\text{sp.reg}} \cap \Omega_k} e'_{j,n} \leq R_{t_n},\ \forall k, \forall n \tag{23}$$

$$e' \geq \underline{0} \tag{24}$$

Convergence to an equilibrium

- Not all the conditions for the existence of an equilibrium solution for the problem are satisfied, in particular the Nikaido-Isoda may not be weakly convex-concave in the whole domain.
- Anyway, the convergence conditions are nearly satisfied, and our case studies have always reached an equilibrium point.

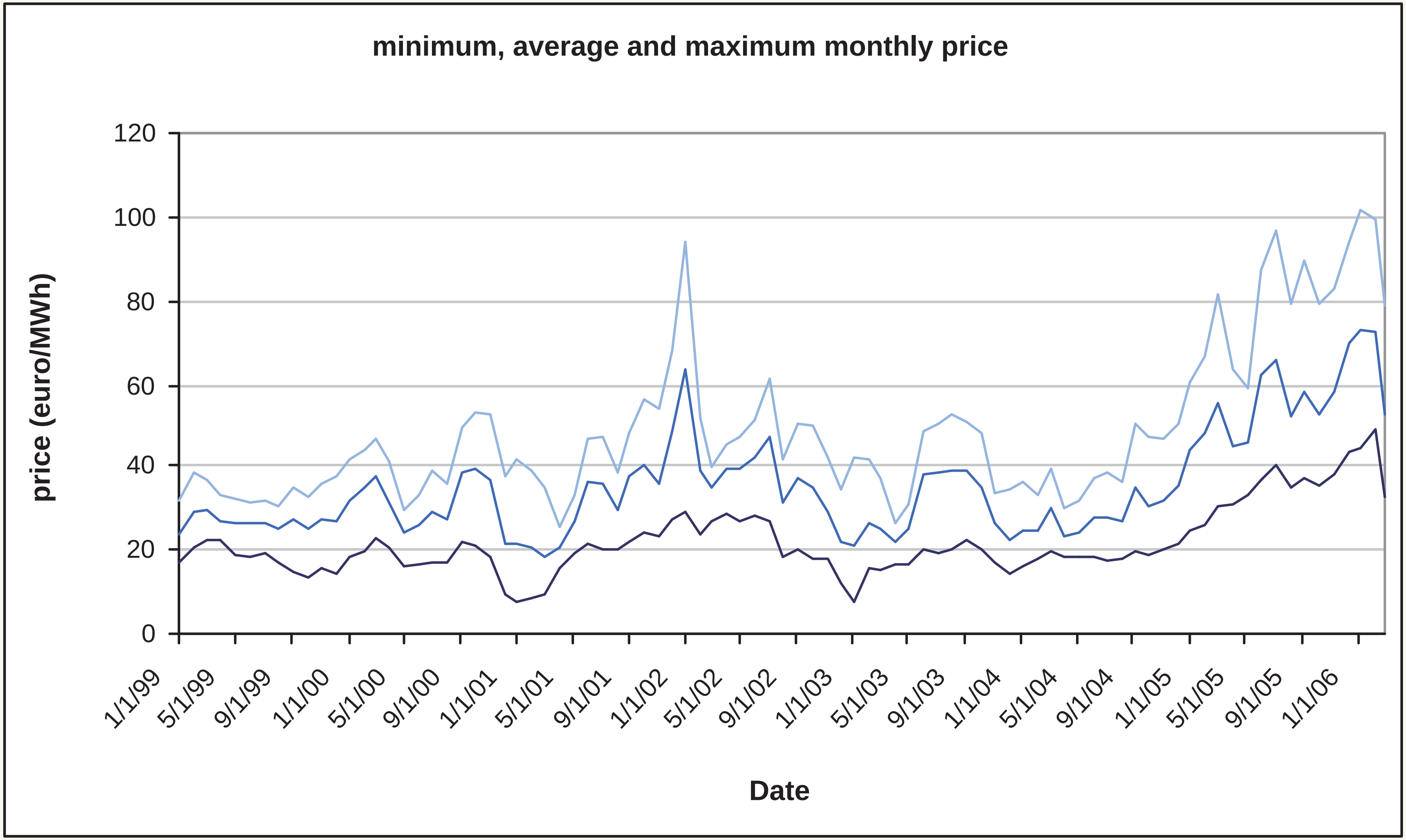
Case Study

- The model is estimated from historical OMEL data
- 50 units belonging to 6 different SGCs are considered.
- A planning horizon of two years is considered and is dived into 11 intervals of variable length

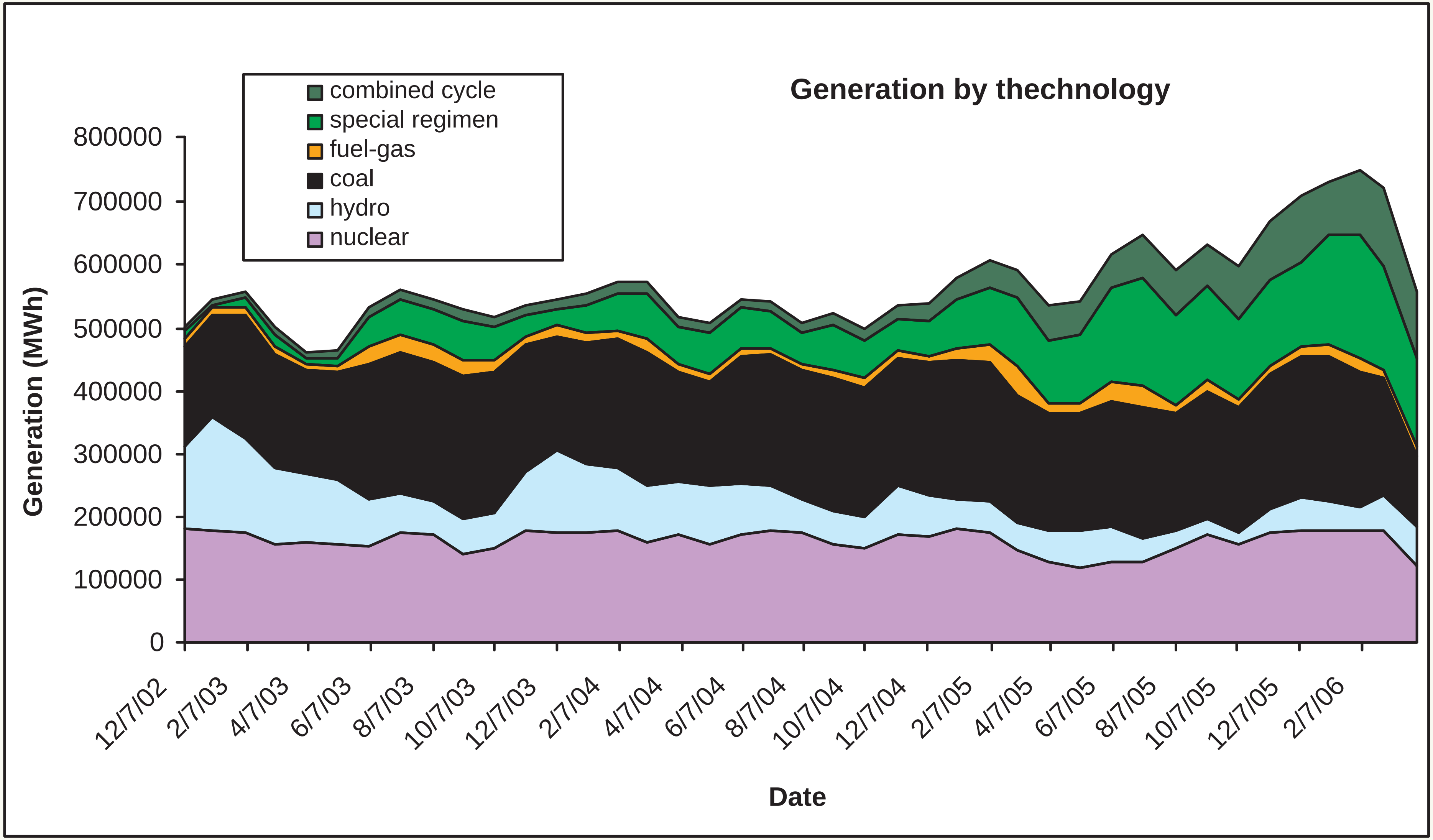
Installed capacity (in %):

	nuclear	hydro	coal	fuel	comb	s.r.	Total
SGC1	0	1.1	6	0	0	0	7
SGC2	14.6	13.7	4	2.9	2.1	1.2	38.5
SGC3	0.8	0.0	0.6	0	2.7	5.6	9.6
SGC4	12.6	7.9	7.9	0	2.0	0.6	30.9
SGC5	0	0.2	2.8	0	0.7	0.1	3.9
SGC6	0.3	3.2	3.8	2.2	0	0.6	10.2
Total	28.2	26.0	25.1	5.2	7.4	8.1	100

Case Study: price historical series



Case Study: generation historical series



Case Study: estimated coefficients

Technology	b^{avg}	b^{dif}
nuclear	-0.0098e-3	0.0636e-3
hydro	-0.0521e-3	0.0850e-3
coal	0.0498e-3	0.0129e-3
fuel	0.3097e-3	0.4719e-3
comb	0.1643e-3	0.1535e-3
special regime	0.0372e-3	-0.1602e-3

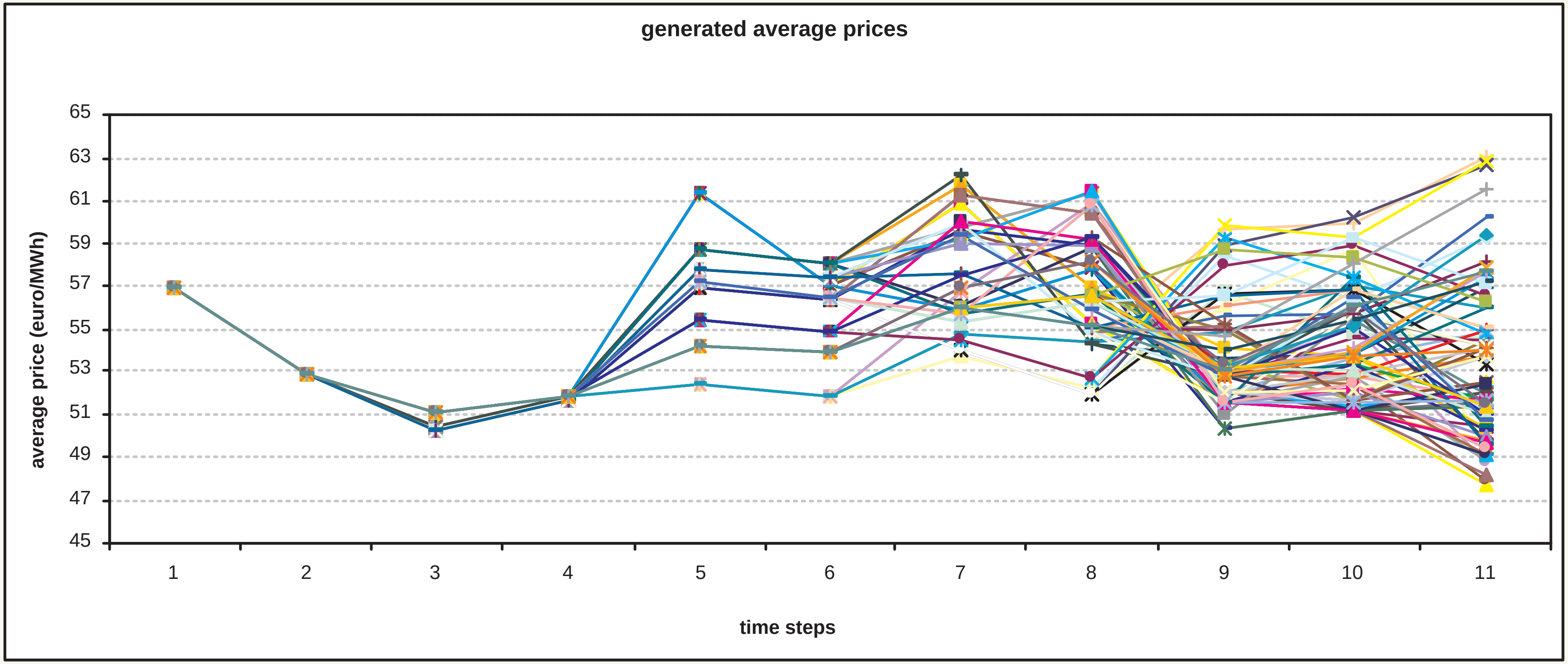
- Nuclear: constant generation implies small influence on the price dynamic
- Hydro: the absence of generation cost produces a distinguishing effect on price
- Fuel: the high generation costs influences positively the price
- Special regimen: generation produces a diminishing of the difference between maximum and minimum price (volatility)

Case Study: computational results

$\lvert\Omega\rvert$	# pl.	scen.	$\lvert\mathcal{N}\rvert$	lmcs	vars	NIRA ite.	cpu time
13	2	1	11	26169(5)	154	8	26 s
13	2	1	11	45045(6)	154	8	43 s
13	2	9	53	135603(5)	742	9	94 s
13	2	9	53	233415(6)	742	9	167 s
50	6	9	53	2850(1)	2703	10	790 s
50	6	9	53	72625(2)	2703	10	2048 s
50	6	81	323	16150(1)	16473	?	? s
50	6	81	323	411825(2)	16473	?	? s

- Parameters:
  - NIRA algorithm $\alpha = 0.95$
  - Environment: AMPL, MINOS, C++ (routines for the calculation of the lmcs' rhs)
  - Platform: Opteron 64 bit, 2.4 GHz

Case Study: generated average price

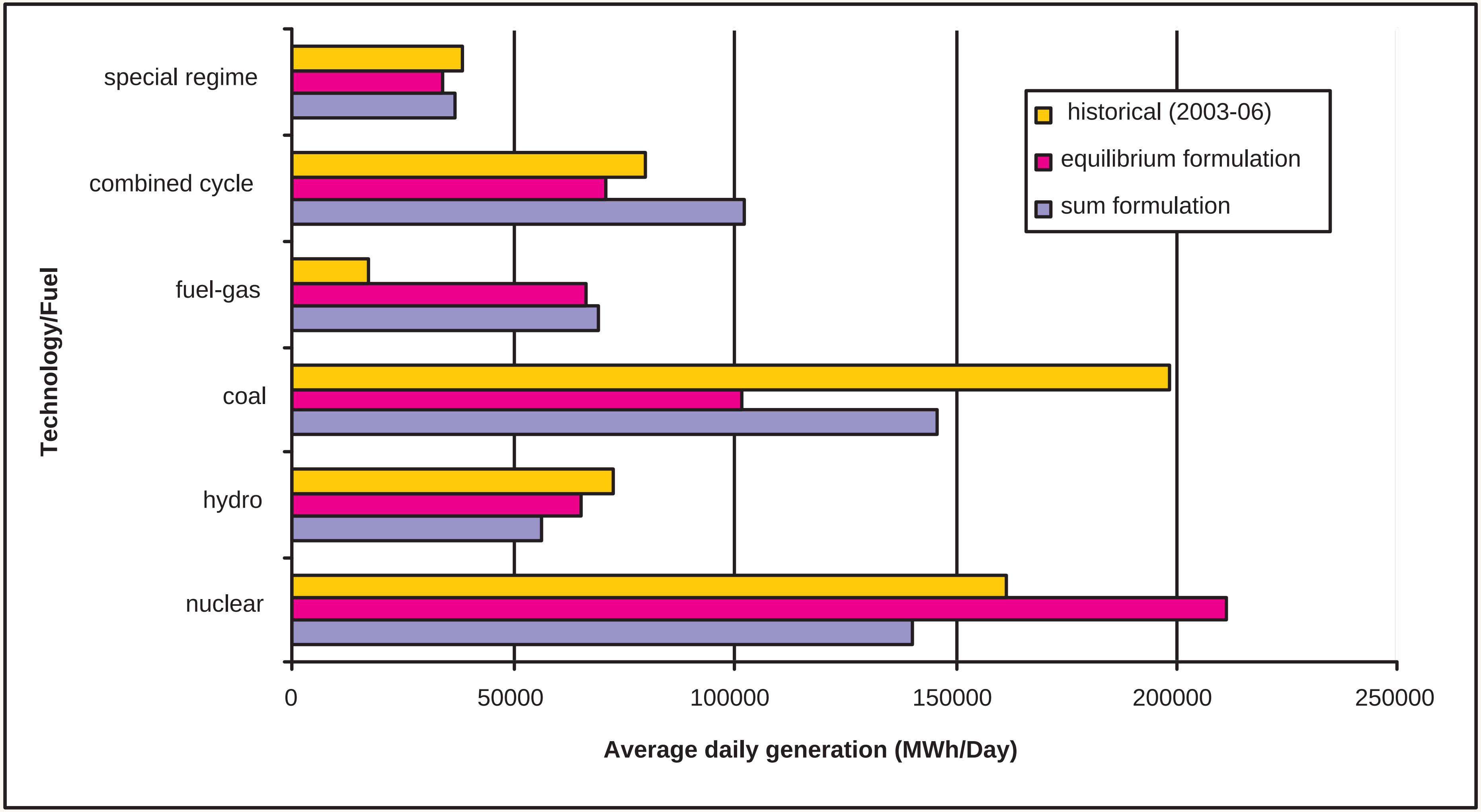


- prices generated by the Nash equilibrium formulation
- the average price series has been generated using only inflows uncertainty (81 scenarios)

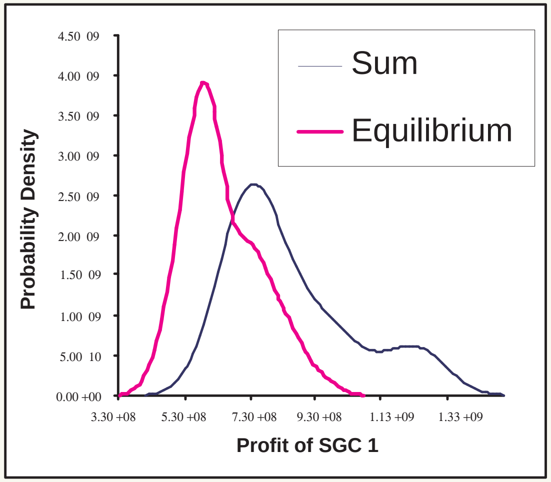
Case Study: sum vs equilibrium

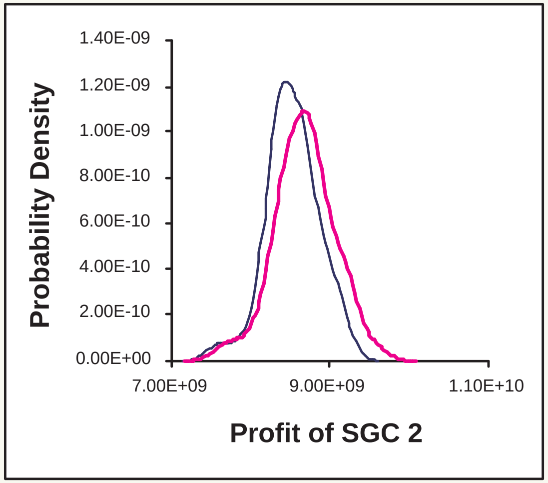
- sum of profit solution generates higher electricity prices and higher profits for some of the generation companies

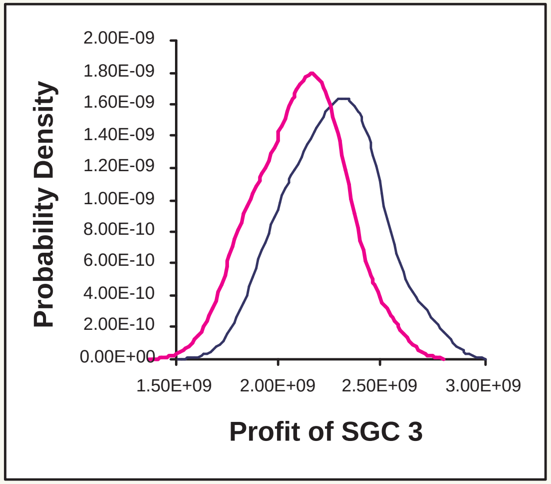
Case Study: sum vs equilibrium (average daily generation)

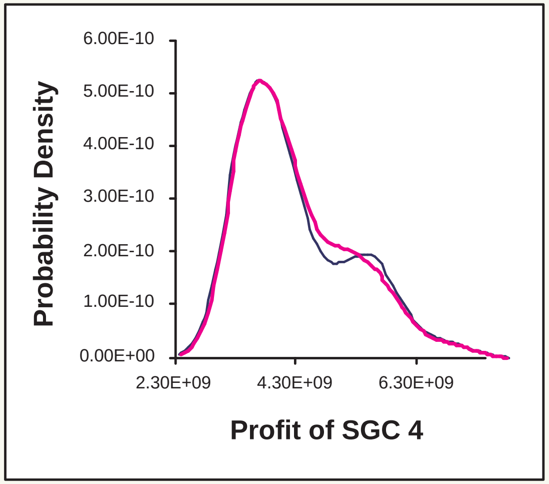


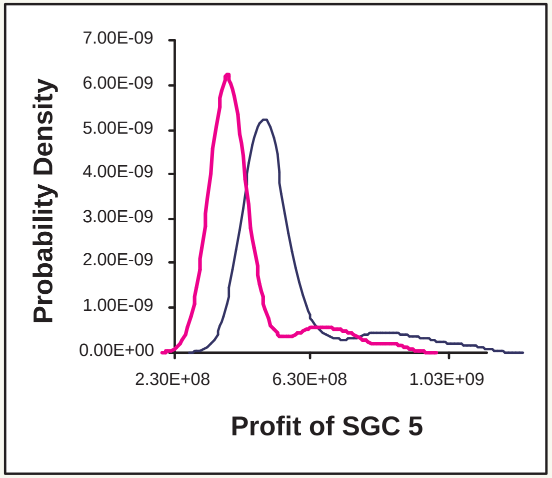
Case Study: sum vs equilibrium (profits' distributions)

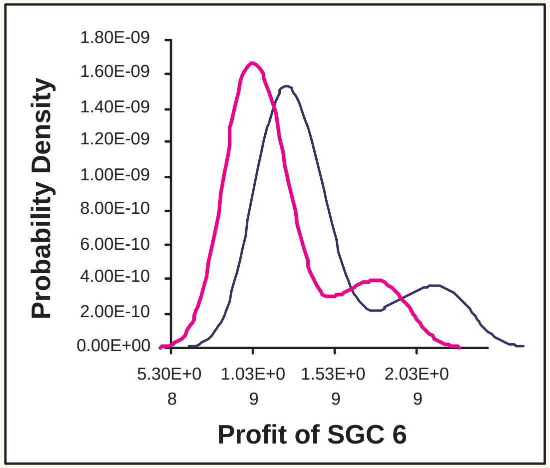












Conclusions

- The model gives an accurate description of the electricity market behavior on the long-term (prices, generation, reserves)
  - Description of the whole market with a granularity of a generation unit
  - Applications can be found both for generation companies and for regulation entities

- Further developments are:
  - Risk handling (CVAR)
  - Futures trading
  - Bilateral contracts consideration
  - Use of better model estimation techniques(eg moment matching)
  - Stochastic fuel prices
  - Emission rights trading